

INTERNATIONAL
ALGEBRAIC CONFERENCE

Book of abstracts of the

INTERNATIONAL
ALGEBRAIC CONFERENCE
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**8th International
Algebraic Conference
in UKRAINE**

*Dedicated to the memory of
Professor Vitaliy Mikhaylovich Usenko*

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SECTION TALK

Testing problems for the asymptotically critical exponential autoregression processes

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Let $\xi^n = (\xi_1, \xi_2, \dots, \xi_n)$, $n \geq 2$ be an observation of the exponential autoregression process $\xi_i = \theta \xi_{i-1} + \varepsilon_i$, $i = 1, 2, \dots$, where $\xi_0 = 0$, $\theta \in (0, \infty)$ is an unknown parameter and $\varepsilon_1, \varepsilon_2, \dots$ are i.i.d. variables which have exponential distribution with density $p(x) = e^{-x}$ when $x \geq 0$ and $p(x) = 0$ when $x < 0$. Denote by \mathbf{P}_θ^n a measure, which gives a distribution of the observation ξ^n . We consider the problem of testing of simple hypotheses H^n and \tilde{H}^n under the observation ξ^n where H^n and \tilde{H}^n mean that a distribution of the observation ξ^n defines by the measures \mathbf{P}_θ^n and $\mathbf{P}_{\tilde{\theta}}^n$ respectively, as $\theta \neq \tilde{\theta}$.

Let $p_\theta(x_1, \dots, x_n)$ be a density of the measure \mathbf{P}_θ^n with respect to the Lebesgue's measure. Introduce the Hellinger integral $H_n(\varepsilon)$ of order $\varepsilon \in (-\infty, \infty)$ for the measures \mathbf{P}_θ^n and $\mathbf{P}_{\tilde{\theta}}^n$ setting [1]

$$H_n(\varepsilon) = H(\varepsilon; \mathbf{P}_\theta^n, \mathbf{P}_{\tilde{\theta}}^n) = \int_0^\infty \dots \int_0^\infty p_\theta^\varepsilon(x_1, \dots, x_n) p_{\tilde{\theta}}^{1-\varepsilon}(x_1, \dots, x_n) dx_1 \dots dx_n.$$

The following theorem about an asymptotical behaviour of the Hellinger integral $H_n(\varepsilon)$ as $n \rightarrow \infty$ is valid.

Theorem 1. Let $\theta_n = 1 - \Delta_n$, $\Delta_n > 0$, $\tilde{\theta}_n = 1 - \tilde{\Delta}_n$, $\tilde{\Delta}_n > 0$ and $\tilde{\Delta}_n = c\Delta_n$, $0 < c < 1$. If θ_n and $\tilde{\theta}_n$ depend on n such that $\Delta_n \rightarrow 0$ and $n\Delta_n \rightarrow \infty$ as $n \rightarrow \infty$. Then for all $\varepsilon \in (-\infty, +\infty)$ there exists limit

$$\lim_{n \rightarrow \infty} n^{-1} \ln H_n(\varepsilon) = \kappa(\varepsilon),$$

where $\kappa(\varepsilon) = -\ln \left(1 + (1 - \varepsilon) \frac{(1-c)}{c} \right)$ for all $\varepsilon \in \left[0, \frac{1}{1-c} \right)$ and $\kappa(\varepsilon) = \infty$ for all $\varepsilon \notin \left[0, \frac{1}{1-c} \right)$.

Introduce the likelihood ratio

$$z_n(x_1, \dots, x_n) = \frac{p_{\tilde{\theta}}(x_1, \dots, x_n)}{p_\theta(x_1, \dots, x_n)}, x_i \in (-\infty, \infty) \text{ for all } i = 1, 2, \dots, n.$$

Let δ_n be a Neyman-Pearson test of level $\alpha_n \in (0, 1)$ for the testing of H^n and \tilde{H}^n under the observation ξ^n . Then (see [1])

$$\delta_n = I(\Lambda_n > d_n) + q_n I(\Lambda_n = d_n),$$

where $I(A)$ is an indicator of the set A , $\Lambda_n = \ln z_n(\xi_1, \dots, \xi_n)$, $d_n \in (-\infty, +\infty)$ and $q_n \in [0, 1]$ are the parameters of the test δ_n , defined by the condition $\mathbf{E}_\theta^n \delta_n = \alpha_n$. Here \mathbf{E}_θ^n means an expectation with respect to the measure \mathbf{P}_θ^n . By β_n we denote 2nd type error probability for the test δ_n .

Then the large deviation theorems are proved for Λ_n as $n \rightarrow \infty$ both under the hypothesis H^n and under the hypothesis \tilde{H}^n . On the basis of large deviation theorems we establish the relation between exponents of the rates of decrease for the error probabilities α_n and β_n of Neyman-Pearson test δ_n as $n \rightarrow \infty$. In this case we use general methods of solution of this problem developed in the papers [2-5] for general binary statistical experiments.

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