Book of abstracts of the

8^{th} International Algebraic Conference in UKRAINE

Dedicated to the memory of
Professor Vitaliy Mikhaylovich Usenko

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July 5–12, 2011, Lugansk, Ukraine

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8th International Algebraic Conference
July 5–12 **(2011)**, Lugansk, Ukraine SECTION TALK

Testing problems for the asymptotically critical exponential autoregression processes

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Let $\xi^n = (\xi_1, \xi_2, \ldots, \xi_n)$, $n \geq 2$ be an observation of the exponential autoregression process $\xi_i = \theta \xi_{i-1} + \varepsilon_i$, $i = 1, 2, \ldots$, where $\xi_0 = 0$, $\theta \in (0, \infty)$ is an unknown parameter and $\varepsilon_1, \varepsilon_2, \ldots$, are i.i.d. variables which have exponential distribution with density $p(x) = e^{-x}$ when $x \geq 0$ and $p(x) = 0$ when $x < 0$. Denote by $\mathbf{P}^n_\theta$ a measure, which gives a distribution of the observation ξ^n. We consider the problem of testing of simple hypotheses H^n and $\tilde{H}^n$ under the observation ξ^n where H^n and $\tilde{H}^n$ mean that a distribution of the observation ξ^n defines by the measures $\mathbf{P}^n_\theta$ and $\mathbf{P}^n_{\tilde{\theta}}$ respectively, as $\theta \neq \tilde{\theta}$.

Let $p_\theta(x_1, \ldots, x_n)$ be a density of the measure $\mathbf{P}^n_\theta$ with respect to the Lebeg's measure. Introduce the Hellinger integral $H_n(\varepsilon)$ of order $\varepsilon \in (-\infty, \infty)$ for the measures $\mathbf{P}^n_{\tilde{\theta}}$ and $\mathbf{P}^n_\theta$ setting [1]

$$H_n(\varepsilon) = H\left(\varepsilon; \mathbf{P}^n_{\tilde{\theta}}, \mathbf{P}^n_\theta\right) = \int_0^\infty \ldots \int_0^\infty p^\varepsilon_{\tilde{\theta}}(x_1, \ldots, x_n)\, p^{1-\varepsilon}_\theta(x_1, \ldots, x_n)\; dx_1 \ldots dx_n.$$

The following theorem about an asymptotical behaviour of the Hellinger integral $H_n(\varepsilon)$ as $n \to \infty$ is valid.

Theorem 1. *Let* $\theta_n = 1 - \Delta_n$, $\Delta_n > 0, \tilde{\theta}_n = 1 - \tilde{\Delta}_n$, $\tilde{\Delta}_n > 0$ *and* $\tilde{\Delta}_n = c\Delta_n, 0 < c < 1$. *If* θ_n *and* $\tilde{\theta}_n$ *depend on* n *such that* $\Delta_n \to 0$ *and* $n\Delta_n \to \infty$ *as* $n \to \infty$. *Then for all* $\varepsilon \in (-\infty, +\infty)$ *there exists limit*

$$\lim_{n\to\infty} n^{-1} \ln H_n(\varepsilon) = \kappa(\varepsilon),$$

where $\kappa(\varepsilon) = -\ln\left(1 + (1-\varepsilon)\frac{(1-c)}{c}\right)$ *for all* $\varepsilon \in \left[0, \frac{1}{1-c}\right)$ *and* $\kappa(\varepsilon) = \infty$ *for all* $\varepsilon \notin \left[0, \frac{1}{1-c}\right)$.

Introduce the likelihood ratio

$$z_n(x_1, \ldots, x_n) = \frac{p_{\tilde{\theta}}(x_1, \ldots, x_n)}{p_\theta(x_1, \ldots, x_n)}, x_i \in (-\infty, \infty) \quad for \quad all \quad i = 1, 2, \ldots, n.$$

Let δ_n be a Neyman-Pearson test of level $\alpha_n \in (0, 1)$ for the testing of H^n and $\tilde{H}^n$ under the observation ξ^n. Then (see [1])

$$\delta_n = \mathrm{I}(\Lambda_n > d_n) + q_n \mathrm{I}(\Lambda_n = d_n),$$

where $\mathrm{I}(A)$ is an indicator of the set A, $\Lambda_n = \ln z_n(\xi_1, \ldots, \xi_n)$, $d_n \in (-\infty, +\infty)$ and $q_n \in [0, 1]$ are the parameters of the test δ_n, defined by the condition $\mathbf{E}^n_\theta\, \delta_n = \alpha_n$. Here $\mathbf{E}^n_\theta$ means an expectation with respect to the measure $\mathbf{P}^n_\theta$. By β_n we denote 2nd type error probability for the test δ_n.

Then the large deviation theorems are prove for Λ_n as $n \to \infty$ both under the hypothesis H^n and under the hypothesis $\widetilde{H}^n$. On the basis of large deviation theorems we establish the relation between exponents of the rates of decrease for the error probabilities α_n and β_n of Neyman-Pearson test δ_n as $n \to \infty$. In this case we use general methods of solution of this problem developed in the papers [2-5] for general binary statistical experiments.

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